Similar Boundary-Layer Analysis for Lee-Surface Heating on Yawed Blunted Cone

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Theme

THE purpose of this paper is to present a simple method for predicting heat transfer on the leeward symmetric plane of a blunted body at a moderate angle of attack. The energy integral for the laminar boundary layer on a plane of symmetry provides a qualitative explanation of the increase of heat transfer in a separated flow region. Using the locally similar solutions of the laminar boundary-layer equation, heat transfer is correlated with the inviscid flow parameters at the boundary-layer edge. This correlation equation was testified by comparing with experimental data for a yawed blunt cone in a hypersonic flow.

Contents

Pressure, heat transfer, and oil flow visualization over the surface of a spherically blunted cone of 15° half-angle were measured at angles of attack $\alpha = 0 \sim 20°$ in the flow of $M_\infty = 7.1$, $p_0 = 15$ atm, and $T_0 = 500°C$ using the NAL 50 cmΦ hypersonic wind tunnel. For these experimental conditions, the three-dimensional inviscid flow was solved numerically using MacCormack's second-order predictor-corrector scheme. Calculated surface pressure agrees well with experimental data, which show a weak circumferential adverse pressure gradient in a separated flow region. Calculated inviscid flow on a wall was compared with the limiting streamlines given by oil streaks. The remarkable secondary flow (cross flow) in the boundary layer was exhibited in the separated region. In Fig. 1, heat transfer data for three different nose radii Rn are compared with DeJarnette's[1] small cross-flow theory. The heat transfer increase is remarkable along the most leeward generator of $\Phi = 0°$. According to Refs. 2 and 3, heat transfer along $\Phi = 0°$ shows a similar boundary-layer behavior if the parabolic similar transformation is applied.

The three-dimensional boundary-layer equations on a plane of symmetry can be written in a streamline coordinate system as

$$(Cf_{\eta\eta})_\eta + (f+\beta_3 Q) f_{\eta\eta} + \hat{\beta}_1 (g - f_\eta^2) = 2\xi (f_\eta f_{\eta\xi} - f_\xi f_{\eta\eta}) \tag{1}$$

$$(CQ_{\eta\eta})_\eta + (f+\beta_3 Q) Q_{\eta\eta} - \hat{\beta}_2 (g - f_\eta^2) - (\beta_1 + \beta_4) f_\eta Q_\eta - \beta_3 Q_\eta^2 = 2\xi (f_\eta Q_{\eta\xi} - f_\xi Q_{\eta\eta}) \tag{2}$$

$$(\hat{C} g_\eta)_\eta + (f+\beta_3) g_\eta - (1-\bar{h}_e)[2\hat{C}(Pr-1) f_\eta f_{\eta\eta}]_\eta = 2\xi (f_\eta g_\xi - f_\xi g_\eta) \tag{3}$$

where

$$\xi = \int (\rho_w \mu_w u_e h_2^2)\, dx \qquad \eta = u_e h_2 (2\xi)^{-0.5} \int \rho\, dz$$

$$f_\eta = u/u_e \qquad Q_\eta = v_y/u_e \qquad g = H/H_e$$

$$h_e = c_p T_e \qquad H = c_p T + 0.5(u^2+v^2) \qquad \hat{C} = C/Pr$$

C = Chapman constant; x, u = length and velocity in streamline direction; y, v = curvilinear coordinate and velocity in cross flow direction; h_2 = metric coefficient of y; z, w = length and velocity normal to the wall. Boundary-layer edge parameters are defined as

$$\beta_1 = \bar{h}_e \hat{\beta}_1 = 2\xi u_{e\xi}/u_e \qquad \beta_2 = \bar{h}_e \hat{\beta}_2 = (p_{yy}/\rho_e u_e^2)\beta_3$$

$$\beta_3 = 2\xi/(h_2 \xi_x) \qquad \beta_4 = \beta_3 h_{2x} \qquad \bar{h}_e = h_e/H_e$$

Boundary conditions are

$$\eta = 0 \quad f = f_\eta = Q = Q_\eta = 0 \quad g = g_w$$

$$\eta \to \infty \quad f_{e_\eta} = g_e = 1 \quad Q_{e_\eta} = 0$$

By integrating the energy equation (3), the heat transfer $\dot{q}$ can be expressed as

$$\dot{q} \propto (\hat{C} g_\eta)_w = \sqrt{2\xi} (\sqrt{2\xi}\,\Theta_f)_\xi + \beta_3 \Theta_Q \qquad \text{nonsimilar form} \tag{4}$$

$$= \Theta_f + \beta_3 \Theta_Q \qquad \text{similar form} \tag{5}$$

where $\Theta_f \equiv \int f_\eta (1-g)\, d\eta$ is the usual streamwise energy thickness, but $\Theta_Q \equiv \int Q_\eta (1-g)$ is the new concept of cross-flow energy thickness.

Equations (1-3) are solved with the local similarity assumption for the wide range of β_1, β_2, β_3, β_4, and $\bar{h}_e$ with $Pr = 0.7$, $C = 1$, and $g_w = 0.3$. From these solutions, the following relations are obtained

$$Q_\eta \propto v_y]_{y=0} \propto v]_{y=dy} < 0 \text{ and } \beta_3 \Theta_Q < 0 \text{ if } \beta_2 \propto p_{yy} > 0$$

$$Q_\eta \propto v_y]_{y=0} \propto v]_{y=dy} > 0 \text{ and } \beta_3 \Theta_Q > 0 \text{ if } \beta_2 \propto p_{yy} < 0 \tag{6}$$

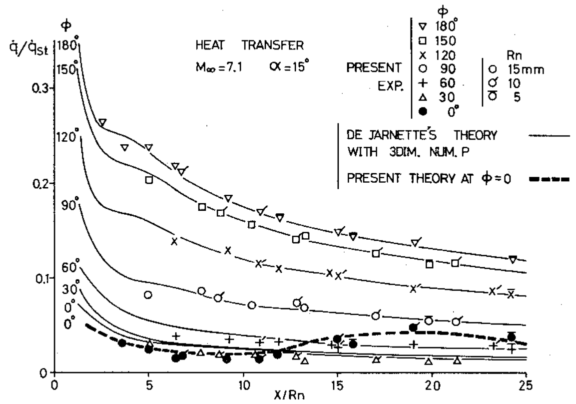

Fig. 1 Heat transfer distributions along cone generators.

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Index categories: Boundary Layers and Convective Heat Transfer – Laminar; Supersonic and Hypersonic Flow.

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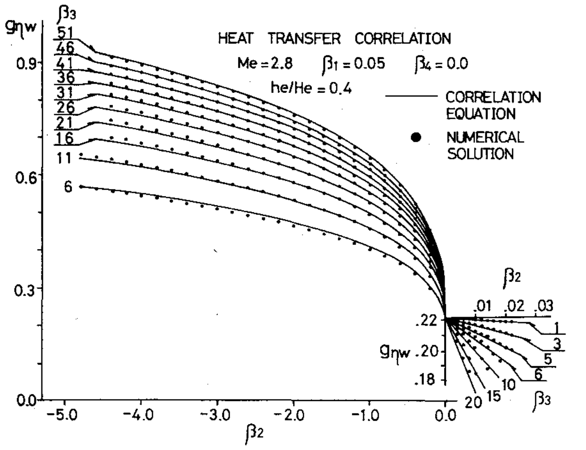

Fig. 2 Similar solutions on a symmetric plane.

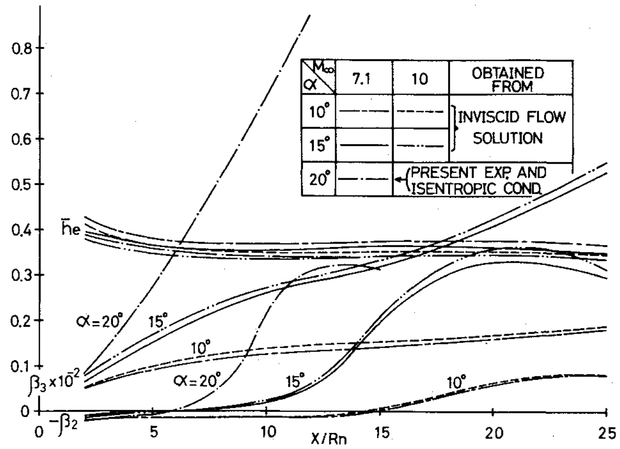

Fig. 4 Boundary-layer edge parameters.

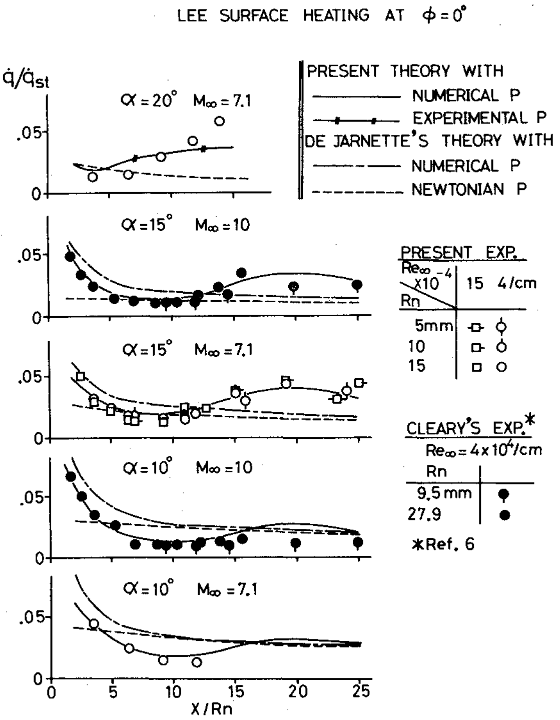

Fig. 3 Heat transfer along the most leeward generator.

These relations agree qualitatively with the experimental result that heat transfer increases along $\Phi = 0°$ in the separated region where the circumferential pressure gradient becomes adverse. Equation (6), of course, does not provide the general condition for incipient separation. The displacement thickness δ^* can be expressed[4] as

$$\delta^* = [\sqrt{2\xi}\int(\rho_e/\rho - f_\eta)\,d\eta - \int\sqrt{2\xi}(h_2\xi_x)^{-1}$$

$$\int Q_\eta d\eta d\xi]/(\rho_e u_e h_2) \qquad (7)$$

This shows that δ^* becomes thin in the separated flow region due to the cross-flow reversal, which makes Q_η positive in Eq. (7). This result agrees with the experiment in Ref. 5 and with the numerical solution in Ref. 3.

The similar solutions of Eqs. (1-3) are shown for $g_{\eta w}$ in Fig. 2. The numerical solutions are correlated by

$$g_{\eta w} = 1/B - (E - 1/B)\beta_2/D \qquad \text{for } 0<\beta_2<0.03$$

$$g_{\eta w} = (1 + A|\beta_2|^N)/(B + C|\beta_2|^N) \qquad \text{for } -5<\beta_2<0 \qquad (8)$$

where

$$E = (1 + A|D|^N)/(B + C|D|^N)$$

$$N = 1/(\beta_3 + 1.4) + 0.445$$

$$B = -1.41\bar{h}_e + 5.04 \qquad C = -\bar{h}_e + 1.8$$

$$D = 0.153/(\beta_3 + 2.44) - 2.8\times10^{-4}$$

$$A = (1 + a\beta_3^n)/(2 + c\beta_3^n) \qquad a = -2.4(\bar{h}_e - 0.75)$$

$$c = -0.085(\bar{h}_e - 1.56) \qquad n = 0.353(\bar{h}_e + 1.124)$$

The effects of β_1 and β_4 on $g_{\eta w}$ are neglected in this correlation equation, which is applicable with 5% accuracy for $|\beta_1|<0.05$, $|\beta_4|<1.0$, $0.3<\bar{h}_e<0.4$, $5<\beta_3<100$ for $-5<\beta_2<0$, and $1<\beta_3<25$ for $0<\beta_2<0.03$. Modifying Lees' formulation from the axisymmetric analogy, heat transfer can be written in the normalized form by the stagnation point value as

$$\dot{q}/\dot{q}_{st} = 0.5(p/p_{st})u_e h_2[(du_e/dx)_{st}$$

$$\int(p/p_{st})u_e h_2^2 dx]^{-0.5}\cdot(g_{\eta w}/g_{\eta w st}) \qquad (9)$$

In Fig. 3 heat transfer calculated by Eqs. (8) and (9) is compared with experimental data. The parameters of β_2, β_3, and $\bar{h}_e$ for these cases are shown in Fig. 4 where β_1 and β_4 changed within the above application limits.

From Fig. 3, it can be concluded that the present cross-flow theory provides good engineering agreement with heat-transfer data in a separated and unseparated region along the most leeward generator.

References

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